Inefficient Credit Booms

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Abstract

This paper studies the welfare properties of competitive equilibria in an economy with financial frictions hit by aggregate shocks. In particular, it shows that competitive financial contracts can result in excessive borrowing ex ante and excessive volatility ex post. Even though, from a first-best perspective the equilibrium always displays under-borrowing, from a second-best point of view excessive borrowing can arise. The inefficiency is due to the combination of limited commitment in financial contracts and the fact that asset prices are determined in a spot market. This generates a pecuniary externality that is not internalized in private contracts. The model provides a framework to evaluate preventive policies which can be used during a credit boom to reduce the expected costs of a financial crisis.

Keywords: Credit market imperfections, constrained efficiency, fire sales, credit booms, corporate hedging.

JEL: E32, E44, E61, G13, G18

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1 Introduction

In the past two decades, both developed and emerging economies have experienced episodes of rapid credit expansion, followed, in some cases, by a financial crisis, with a collapse in asset prices, credit and investment.[1] This experience has led policy makers to be increasingly wary of credit booms and to propose various preventive measures to reduce the probability and/or the depth of a potential crisis.[2] However, relatively little theoretical work has analyzed the reasons why a credit boom may be inefficient from an ex ante perspective, and whether any intervention is warranted. If the private sector correctly perceives the risk of a negative aggregate shock, it will incorporate this risk in its optimal decisions. If agents still decide to borrow heavily during the boom, it means that the expected gains from increased investment today more than compensate for the expected costs of financial distress in the future. Therefore, to assess the need for policy intervention, one needs to understand how, and under what conditions, this private calculation leads to inefficient decisions at the social level. In this paper, I address this question focusing on a pecuniary externality which arises from the combination of financial constraints with a competitive market for real assets. I analyze constrained efficiency by considering a planner who faces the same constraints faced by the private economy, and asking whether a reduction in borrowing ex ante can lead to a Pareto improvement. My main result is that excessive borrowing can arise in equilibrium, and that it is associated to an excessive contraction in investment and asset prices if the crisis takes place.

The paper develops a three-period model of investment with financial frictions. In the first period, entrepreneurs with limited internal funds borrow and invest in some productive asset (real estate, machinery, equipment, etc.). In the second period, their revenues are subject to an aggregate shock, which can take two values, good and bad. When the bad shock hits, they face operational losses. Given their limited access to outside funds, they need to sell part of the assets to finance these losses. Assets are sold on a competitive market, where they are absorbed by a traditional sector, which makes a less productive use of them. Each entrepreneur has access to state-contingent debt contracts: he can decide both how much to borrow in the first period and how much to repay in different states of the world in the following periods. By investing more in the first period the entrepreneur earns higher revenues if the good shock

[1] For the main stylized facts on boom-bust cycles see Gourinchas, Valdes and Landerretche (2001), Borio and Lowe (2002), Bordo and Jeanne (2002), Tornell and Westermann (2002), Ranciere, Tornell and Westermann (2003).

[2] See Borio (2003) and references therein.

is realized, but faces larger losses if the bad shock hits. Entrepreneurs are fully rational and correctly perceive the risks and rewards associated to different financial decisions. However, since they are atomistic, they do not take into account the general equilibrium effect of asset sales on prices. This is the pecuniary externality at the basis of my inefficiency result. By reducing aggregate investment ex ante a planner can reduce the size of the asset sales in the bad state. This increases asset prices, leading to a reallocation of funds from the traditional sector, who is buying assets, to the entrepreneurial sector, who is selling them. Due to the presence of financial frictions, this reallocation leads to an aggregate welfare gain, which is not internalized by private agents.

Many accounts of recent financial crises have emphasized the interaction between asset prices and financial distress in the corporate and financial sector. As an example, take the case of the banking sector in Thailand prior to the crisis of 1997. In the first half of the 90s Thai banks increased their investment in real estate, both directly, through loans to property developers, and indirectly, through loans to finance companies which had extensive investment in real estate. When the crisis erupted, the fall in real estate prices eroded the value of the assets held by the banks, as loans, backed by real estate guarantees, started going into default. This prompted a cut-back in lending, which, in turns, led to a further reduction in the demand for real estate and a further drop in real estate prices. In these circumstances, the large supply of recently developed real estate, fueled by bank lending during the boom, contributed to the severe collapse in prices during the crisis.[3] This is the type of mechanism I model in this paper. To capture the essence of the argument, I do not model explicitly financial intermediation and I concentrate on a setup where financially constrained agents invest directly in real assets.

Current policy debates mention a number of reasons why a credit boom might be inefficient: irrational optimism of the borrowers; moral hazard caused by the expectation of a bailout; inefficient delays in the treatment of information; some negative externality by which higher borrowing of some agents may increase "systemic risk." Of these arguments, only the first two have been fully developed in the literature.[4] This paper attempts to formalize the "systemic risk" argument, focusing on a pecuniary externality working through asset prices. The idea

[3] See Herring and Wachter (1999) for a narrative of the boom-bust cycle in Thailand. See Watanabe (2007) for an empirical estimate of the effect of losses on real estate investment on banks' lending in Japan.

[4] The literature on optimal monetary policy has analyzed economies where an investment boom is driven by an irrational fad, or "bubble," see Cecchetti, Genberg, Lipsky and Wadhwani (2000), Bernanke and Gertler (2001), and Dupor (2002). For the moral hazard argument applied to recent crises, see McKinnon and Pill (1996), Corsetti, Pesenti and Roubini (1999), Tornell and Schneider (2004).

of focusing on the general equilibrium feed-back between financial distress and asset prices goes back to Shleifer and Vishny (1992) and Kiyotaki and Moore (1997). The role of asset "fire sales" during recent episodes of financial crisis has been emphasized by Krugman (1998). Systematic evidence on fire sales is presented in Pulvino (1998) and Aguiar and Gopinath (2005).

The fundamental source of inefficiency in this paper is in financial frictions, both on the borrowers' (entrepreneurs) and on the lenders' (consumers) side. In particular, my model assumes that both entrepreneurs and consumers have limited ability to commit to future repayments. Lack of commitment on the entrepreneurs' side implies that they have limited access to external finance. Lack of commitment on the consumers' side limits the entrepreneurs' ability to insure ex ante against aggregate liquidity shocks, as in Holmstrom and Tirole (1998). As I will show in Section 4, the combination of these two imperfections drives the inefficiency result.

The paper is related to the large literature on the role of financial frictions in the amplification and propagation of macroeconomic shocks.[5] Existing papers have compared the equilibrium arising in models with financial constraints with a first-best benchmark in which no financial constraints are present. The main contribution of this paper is to study welfare from a second-best perspective and to identify the possibility of over-borrowing. The closer precedent to the model presented is Krishnamurthy (2003), who develops a model à la Kiyotaki and Moore (1997) with state-contingent contracts. He uses the model to argue that, in presence of state-contingent contracts, the degree of amplification is smaller than in the case of non-state-contingent debt.[6] Gertler (1992) offers an early analysis of multi-period financial contracts in an environment with agency costs, aggregate shocks, and state-contingent contracts. The analysis of state-contingent debt is also related to the literature on hedging in the presence of financial constraints. In particular, Froot, Scharfstein and Stein (1993) make the case that firms with access to costly external finance and with a concave technology should hedge cash-flow shocks. In my model firms have a constant returns to scale technology. However, a similar motive for hedging aggregate cash-flow shocks arises in general equilibrium.

[5] See Bernanke and Gertler (1989), Lamont (1995), Carlstrom and Fuerst (1997), Acemoglu and Zilibotti (1997), Aghion, Banerjee and Piketty (1999), Tornell and Schneider (2003), Rampini (2003), Cooley, Marimon and Quadrini (2004), Guerrieri and Lorenzoni (2007) and references in Bernanke et al. (2001).

[6] "Fire sales" of assets are not present in his model, i.e. entrepreneurial investment is always positive. Therefore, my conjecture is that over-borrowing cannot arise in that setup, although the equilibrium is not constrained efficient.

Since asset prices drop when aggregate entrepreneurial wealth is low, that increases the rate of return on investing in the bad state, and induces entrepreneurs to transfer financial resources to that state.

From a methodological standpoint, the idea that the competitive equilibrium in economies with endogenous borrowing constraints can be constrained inefficient goes back to Kehoe and Levine (1993). They show that in an economy with limited enforcement the first welfare theorem holds when there is only one good, but fails to hold with more than one good.[7] In the second case, private contracts fail to internalize their effect on equilibrium prices, and, in turns, these prices affect the financial constraints. This paper shows that pecuniary externalities of this type provide a useful framework to study credit booms. Recent contributions that use constrained efficiency analysis to study the role of preventive policies in financial markets include Caballero and Krishnamurthy (2001, 2003), Lorenzoni (2001), Allen and Gale (2004), Gai, Kondor and Vause (2006), and Farhi, Golosov and Tsyvinski (2007).

A recent paper by Bordo and Jeanne (2002) approaches credit booms from a point of view similar to the one taken here, focusing on the trade-off between high investment ex ante and financial distress ex post. They consider an economy with sticky prices and show that, if the firms are highly leveraged when a negative shock hits, this causes a sharper reduction in investment and output. In this environment they study the effect of preventive monetary policy, which can help to reduces firms' leverage ex ante.

The paper is organized as follows. In Section 2, I introduce the model. In Section 3, I characterize the competitive equilibrium. Section 4 contains the welfare analysis and a discussion of policy implications. Section 5 concludes. All the proofs are in the appendix.

2 The Model

There are three periods, $0, 1$ and 2, and two groups of agents of equal mass, consumers and entrepreneurs. There are two goods, a perishable consumption good and a capital good. Consumption goods can be turned into capital goods one for one at any point in time, but the opposite is not feasible.

Consumers are risk neutral with preferences represented by the utility function $E[c_0 + c_1 + c_2]$, and receive a constant endowment e of consumption goods in each period. Entrepreneurs are

[7] In turns, this result is related to the inefficiency result in economies with incomplete markets, Geanakoplos and Polemarchakis (1986).

also risk neutral but only consume in period 2. Their preferences are given by $E\left[c_2^e\right]$. They begin life with an endowment n of consumption goods and receive no further endowment in the following periods. Moreover, they have access to the following technology. In period 0, they choose the level of investment k_0. In period 1, this investment yields $a_s k_0$ units of consumption good, with $a_s > 0$. The productivity of investment at date 1, a_s, is random and depends on the aggregate state s, which takes the values l and h (low and high) with probabilities π_l and π_h. In period 1 the capital k_0 requires maintenance in order to remain productive. Maintenance costs are equal to γ units of consumption goods per unit of capital. If γ is not paid, capital is scrapped, i.e., it fully depreciates. Entrepreneurs choose the fraction of capital they want to keep productive, denoted by $\chi_s \in [0,1]$. Hence, $\chi_s k_0$ is the undepreciated part of the capital stock and total maintenance costs are equal to $\gamma\chi_s k_0$. At the end of period 1, entrepreneurs choose the capital stock for next period, k_{1s}, by making the net investment $k_{1s} - \chi_s k_0$. The capital stock k_{1s} produces Ak_{1s} units of consumption goods in period 2, with $A > 1$. Capital fully depreciates at the end of period 2.

Each consumer owns a firm in the "traditional sector." Firms in the traditional sector invest capital k_{1s}^T in period 1 to produce consumption goods in period 2. The technology of the traditional sector is represented by the production function $F(k_{1s}^T)$. The function $F(.)$ is increasing, strictly concave, twice differentiable, and satisfies the following properties: $F(0) = 0$, $F'(0) = 1$, $F'\left(k_{1s}^T\right)$ is bounded below, with lower bound $\underline{q}$.

The goods and capital markets are competitive. The price of capital in period 1 is denoted by q_s. For simplicity, I assume that the economy begins with no capital, so the price of capital is one in period 0, as long as some investment takes place. On the other hand, the price of capital is zero in period 2, since that is the final date.

2.1 Financial contracts with limited commitment

At date 0, entrepreneurs offer financial contracts to consumers. A financial contract specifies a loan d_0 at date 0 from the consumer to the entrepreneur and state-contingent payments d_{1s} and d_{2s} from the entrepreneur to the consumer in periods 1 and 2, for each state s.

In period 0, the entrepreneur can invest his initial wealth plus the amount borrowed from the consumer,

$$k_0 \leq n + d_0.$$

In period 1, the entrepreneur's cash flow is equal to current revenues minus maintenance costs.

Part of these funds are used to pay d_{1s} to the consumer, the rest goes to finance current investment. The budget constraint is then

$$q_s\left(k_{1s}-\chi_s k_0\right)\leq a_s k_0-\gamma\chi_s k_0-d_{1s}.$$

Finally, in period 2, the entrepreneur can consume the final revenues net of debt repayments,

$$c^e_{2s}\leq Ak_{1s}-d_{2s}.$$

The consumer's budget constraints are easily derived. If he accepts the contract, his expected utility is

$$e-d_0+\sum_s\pi_s\left(e+d_{1s}-q_s k^T_{1s}+e+d_{2s}+F\left(k^T_{1s}\right)\right), \tag{1}$$

while, if he does not accept, it is

$$e+\sum_s\pi_s\left(e-q_s k^T_{1s}+e+F\left(k^T_{1s}\right)\right).$$

I will assume throughout the paper that e is sufficiently large that the non-negativity constraints for c_0, c_{1s} and c_{2s} are never binding.[8] The consumer's participation constraint is then given by

$$d_0\leq\sum_s\pi_s\left(d_{1s}+d_{2s}\right).$$

Financial contracts are subject to a form of limited commitment, both on the entrepreneur's and on the consumer's side. Consider first the entrepreneur. In periods 1 and 2 he chooses whether or not to make the contractual payments d_{1s} and d_{2s}. If he fails to pay, he gets to make a take-it-or-leave-it offer to the consumer regarding current and future payments. If the consumer rejects the offer, the firm is liquidated. When the firm is liquidated a fraction $(1-\theta)$ of the firm's current profits is lost, where θ is a scalar in $(0,1)$. The rest of the profits and the firm's capital stock go to the consumer. Therefore, if liquidation occurs in period 1, the consumer receives the revenue $\theta a_s k_0$ and the capital stock k_0. The latter can be either scrapped or sold on the asset market after paying the maintenance costs.[9] Given that the price of capital

[8] Given the assumptions made below, a sufficient condition for this is

$$e>\frac{1}{1-\sum_s\pi_s\left(\theta a_s+1-\gamma\right)}n.$$

This ensures that the consumer is neither constrained in period 0, when choosing whether to accept the financial contract, nor in period 1, when choosing how much to invest in the traditional sector.

[9] The consumer might also use some of this capital in his traditional firm. Since there is a competitive market for capital, this option is irrelevant. Notice also that, in the event of a default, the entrepreneur has no incentive to pay the maintenance cost.

is q_s, the consumer will pay the maintenance costs as long as $q_s - \gamma > 0$. Therefore, the net value of a liquidated firm in period 1 is $\left(\theta a_s + (q_s - \gamma)_+\right) k_0$. From now on, the notation $(.)_+$ will be used to denote the non-negative part of a variable, e. g., $(q_s - \gamma)_+ \equiv \max\{q_s - \gamma, 0\}$. In period 2, the value of a liquidated firm is simply $\theta A k_{1s}$. Since state-contingent contracts are available, I can, without loss of generality, restrict attention to contracts where default and renegotiation never happen in equilibrium. The entrepreneur will never default if and only if the following inequalities are satisfied

$$d_{1s} + d_{2s} \leq \left(\theta a_s + (q_s - \gamma)_+\right) k_0, \tag{2}$$

$$d_{2s} \leq \theta A k_{1s}, \tag{3}$$

for $s = l, h$. A natural interpretation of these constraints is that the liquidation value of the firm acts as collateral for the financial obligations of the entrepreneur.

The consumer can always walk away from a financial contract and his current and future income cannot be seized. Then, the consumer no-default conditions are

$$d_{1s} + d_{2s} \geq 0, \tag{4}$$

$$d_{2s} \geq 0, \tag{5}$$

for $s = l, h$. Note that the revenue of the traditional sector, $F\left(k_{1s}^T\right)$, cannot be used as collateral in financial contracts.

For simplicity, I consider only bilateral financial contracts involving one entrepreneur and one consumer, which, in the current environment, is without loss of generality. In particular, cross-holdings of financial securities across entrepreneurs are irrelevant, given that there is only aggregate uncertainty.[10]

Three additional assumptions will be useful in the analysis. First, I assume that the liquidation value of entrepreneurial firms is sufficiently small.

Assumption A *The parameter θ is small enough that the following inequalities hold*

$$\sum_s \pi_s \left(\theta a_s + 1 - \gamma\right) < 1,$$
$$\theta A < 1.$$

[10] See Holmstrom and Tirole (1998) for a thorough discussion of this issue in a related model.

This will imply that investment cannot be fully financed with outside funds either in period 0 or 1. Second, I impose some restrictions on the shocks a_l and a_h, and on the maintenance cost γ.

Assumption B *The values of a_l and a_h are such that*

$$\begin{aligned}(1-\theta)\, a_h + \theta A - 1 &> 0, \\ a_l + \theta A - \gamma &< 0.\end{aligned}$$

The maintenance cost γ satisfies $\gamma < \underline{q}$.

The first two conditions will be used to show that entrepreneurs' investment is positive in the high state and negative in the low state. The last condition allows me to rule out scrapping of capital in equilibrium. Finally, the next assumption simplifies the analysis by ruling out multiple equilibria in the asset market at date 1.

Assumption C *The function $\left(F'\left(k^T\right) - \theta A\right) k^T$ is increasing in k^T.*

2.2 Equilibrium definition

The entrepreneur's individual problem is to choose a financial contract and investment and consumption levels so as to maximize expected utility subject to the budget constraints, the consumer's participation constraint, and the no-default constraints introduced above. The consumer's problem is to choose which financial contract to accept, if any, and, then, set consumption and investment in the traditional firm so as to maximize expected utility subject to the budget constraints. Both the entrepreneur and the consumer take as given the vector of asset prices $\{q_s\}$.

A symmetric competitive equilibrium is given by a vector of asset prices $\{q_s\}$, a financial contract $\langle d_0, \{d_{1s}, d_{2s}\}\rangle$, investment and consumption decisions for the entrepreneur k_0 and $\{\chi_{1s}, k_{1s}, c^e_{2s}\}$, and investment and consumption decisions for the consumer c_0 and $\left\{k^T_{1s}, c_{1s}, c_{2s}\right\}$, such that entrepreneurs' and consumers' behavior are optimal, and goods and capital markets clear in all periods and states.

3 Equilibrium

In this section, I give a characterization of the equilibrium. First, I will look at the optimal borrowing and investment decisions of the entrepreneur for given asset prices $\{q_s\}$. Next, I will show how the entrepreneurs' aggregate behavior affects the asset prices which clear the capital market in period 1. Finally, I will put the two pieces together and show how equilibrium borrowing and investment are determined. It is useful, however, to begin with a preliminary result regarding the capital market in period 1.

Lemma 1 *In equilibrium asset prices are characterized by the conditions*

$$\begin{aligned} q_s &= F'\left(k_{1s}^T\right), \\ k_{1s}^T &= \left(k_0 - k_{1s}\right)_+, \end{aligned}$$

for $s = l, h$, *and no scrapping occurs in equilibrium,* $\chi_s = 1$ *for* $s = l, h$.

From this lemma it follows that two cases are possible in period 1 capital market. In the first case, the price of capital is one, the traditional sector chooses zero investment (recall that $F'(0) = 1$), and the entrepreneurial sector makes positive investment $k_{1s} - k_0 > 0$. This investment is done by transforming consumption goods into capital goods. In the second case, the price of capital is smaller than one, no consumption goods are transformed into capital, and the entrepreneurial sector sells capital to the traditional sector. In this case market clearing requires $k_{1s}^T = k_0 - k_{1s}$, and optimality for firms in the traditional sector requires $q_s = F'\left(k_{1s}^T\right)$.

This result implies that equilibrium asset prices are bounded

$$\underline{q} \leq q_s \leq 1. \tag{6}$$

Since $\underline{q} > \gamma$ by Assumption B, this also implies that $q_s - \gamma > 0$ which rules out scrapping of capital in equilibrium. Both properties will help in the characterization of optimal financial contracts.

3.1 Optimal financial contracts

Since scrapping is never optimal, I simplify the notation by defining net profits per unit of capital

$$x_s = a_s - \gamma.$$

Moreover, I describe the financial contract in terms of the net present value of promised repayments per unit of capital, which are given by[11]

$$b_{1s} = \frac{d_{1s} + d_{2s}}{k_0},$$

[11] In the proof of Lemma 2, I show that the optimal values of k_0 and k_{1s} are positive, so these ratios are well defined.

and

$$b_{2s} = \frac{d_{2s}}{k_{1s}}.$$

The entrepreneur's problem can be written in the following form. The entrepreneur chooses the financial contract $\langle d_0, \{b_{1s}, b_{2s}\}\rangle$ and the investment levels k_0 and $\{k_{1s}\}$ to maximize

$$\sum_s \pi_s \left(A - b_{2s}\right) k_{1s}, \tag{7}$$

subject to the budget constraints

$$k_0 \leq n + d_0, \tag{8}$$

$$q_s k_{1s} \leq \left(q_s + x_s - b_{1s}\right) k_0 + b_{2s} k_{1s} \quad \text{for } s = l, h, \tag{9}$$

the no-default constraints

$$0 \leq b_{1s} \leq \theta a_s + q_s - \gamma \quad \text{for } s = l, h, \tag{10}$$

$$0 \leq b_{2s} \leq \theta A \quad \text{for } s = l, h, \tag{11}$$

the consumer participation constraint

$$d_0 \leq \sum_s \pi_s b_{1s} k_0, \tag{12}$$

and non-negativity constraints for k_0 and k_{1s}. The following lemma gives a characterization of optimal financial contracts.

Lemma 2 *Given a vector of equilibrium prices $\{q_s\}_{s=l,h}$, an individually optimal financial contract satisfies the conditions*

$$b_{1s} = 0 \qquad \text{if } z_0 < z_{1s}, \tag{13a}$$

$$b_{1s} \in [0, \theta a_s + q_s - \gamma] \qquad \text{if } z_0 = z_{1s}, \tag{13b}$$

$$b_{1s} = \theta a_s + q_s - \gamma \qquad \text{if } z_0 > z_{1s}, \tag{13c}$$

$$b_{2s} = \theta A,$$

for $s = l, h$, where

$$z_{1s} = \frac{(1-\theta) A}{q_s - \theta A}, \tag{14}$$

and

$$z_0 = \frac{\sum_s \pi_s z_{1s} \left(q_s + x_s - b_{1s}\right)}{1 - \sum_s \pi_s b_{1s}}. \tag{15}$$

The variables z_0 and z_{1s} defined in (14)-(15) are the Lagrange multipliers on the budget constraints at dates 0 and 1, they represent the *rates of return on entrepreneurial wealth* in periods 0 and 1.[12] Since they play an important role in the analysis to follow, let me provide some intuition for them. When investing in period 1, the entrepreneur can buy capital at the price q_s and finance this investment by borrowing θA per unit of capital. One dollar of internal funds can thus be leveraged by a factor of $1/(q_s - \theta A)$. At date 2 this investment gives A per unit of capital, of which θA is paid to consumers. Therefore, the marginal return on internal funds available in period 1 is $z_{1s} = (1-\theta) A/(q_s - \theta A)$. Going back to period 0, one extra dollar of internal funds at date 0 can be leveraged by a factor of $1/(1 - \sum_s \pi_s b_{1s})$ and the capital invested gives a random net payoff of $q_s + x_s - b_{1s}$ in period 1. This net payoff can then be reinvested at the rate of return z_{1s}. Averaging across states gives expression (15).

The choice of the repayment ratios $\{b_{1s}\}$ depends on the comparison of rates of return on internal funds in periods 0 and 1, state by state. Suppose the entrepreneur increases his borrowing in period 0 by π_s dollars by promising one dollar in period 1 in state s. The increase in funds available at date 0 increases the entrepreneur's utility by $\pi_s z_0$, while the decrease in funds available at date 1 decreases the entrepreneur's utility by z_{1s} with probability π_s. Comparing these marginal effects shows that as long as $z_0 > z_{1s}$ the entrepreneur will increase his promised repayments in state s, up to the point where $b_{1s} = \theta a_s + q_s - \gamma$. If, instead, $z_0 < z_{1s}$ the entrepreneur will decrease his promised repayments until $b_{1s} = 0$. An interior choice for b_{1s} will only arise if $z_0 = z_{1s}$.

The choice of the repayments in period 2 is much simpler. The marginal utility of entrepreneurial wealth is always equal to one in period 2, since at that point the entrepreneur can only consume. Given that $z_{1s} > 1$ in all states, this implies that the entrepreneur always commits to maximum repayments in period 2, $b_{2s} = \theta A$, in order to maximize investment in period 1.

The argument above shows that the optimal financial contract depends on the prices q_s through their effect on the rates of return z_{1s} and z_0. In turns, the prices q_s depend on the contracts chosen by the entrepreneurs, since they determine how much capital they sell on date 1. I will now look at this relation, before turning to general equilibrium.

[12] More precisely, z_{1s} is the Lagrange multiplier normalized by the probability π_s.

3.2 Asset prices

Consider the asset market in period 1, taking as given the financial contract chosen by the entrepreneurs. Net investment by the entrepreneurs is

$$k_{1s} - k_0 = \frac{x_s + \theta A - b_{1s}}{q_s - \theta A} k_0 \tag{16}$$

for $s = l, h$. This expression comes from rearranging (9) and using the result that the financial constraint is always binding in period 2 (from Lemma 2). It is not difficult to show that in the high state the right-hand side of (16) is positive and so is investment. The opposite happens in the low state.[13] Then, Lemma 1 implies that $q_h = 1$ and $q_l < 1$. Therefore, let me focus on the determination of the asset price in the low state.

In the low state, the entrepreneurial firm is facing net losses, since $x_l k_0 < 0$. Due to the collateral constraint, the firm has limited ability to borrow against future income. If it tried to keep the existing capital stock unchanged, its borrowing capacity would be insufficient to cover current losses, since $b_{2l} k_0 \leq \theta A k_0 < -x_l k_0$ (the first inequality follows from no default, the second from Assumption B). Moreover, the firm has limited ability to buy insurance ex ante, due to consumers' limited commitment, $b_{1l} \geq 0$. The only remaining option to cover the firm's losses is to sell part of the capital stock. To induce the traditional sector to absorb this capital the price of capital has to fall below 1.

Figure 1 gives a graphical illustration of the equilibrium in the low state, for given values of k_0 and b_{1l}. Curve S plots the entrepreneurs' supply of capital as a function of q_l. For completeness, the figure includes the regions where $q_l \geq A$ and $q_l \leq \gamma$, although such prices never arise in equilibrium. When $\gamma < q_l < A$ the entrepreneurs' behavior is captured by (16) and the supply of capital is given by $-\left(x_l + \theta A - b_{1l}\right) k_0 / \left(q_l - \theta A\right)$. Notice that in this region the supply is decreasing in q_l: a price increase allows entrepreneurs to sell a smaller amount of capital to cover their losses.[14] When q_l goes above A, entrepreneurial investment becomes unprofitable and entrepreneurs sell all the capital stock k_0. Finally, when q_l goes below γ scrapping is optimal and entrepreneurial capital is destroyed. In the same figure, I plot the traditional sector demand for capital, described by the condition $q_l = F'\left(k_0 - k_{1l}\right)$.

[13] See Lemma 4 in the Appendix.

[14] The fact that the supply is decreasing has two implications: it magnifies the effect of entrepreneurial losses on asset prices, and it opens the door to multiple equilibria. The amplification is important because it increases the quantitative relevance of the pecuniary externality discussed in Section 4. Multiplicity is ruled out in this paper, by virtue of Assumption C. Gai, Kapadia, Millard and Perez (2006) study the implications of a similar model, focusing on the case where multiple equilibria are possible.

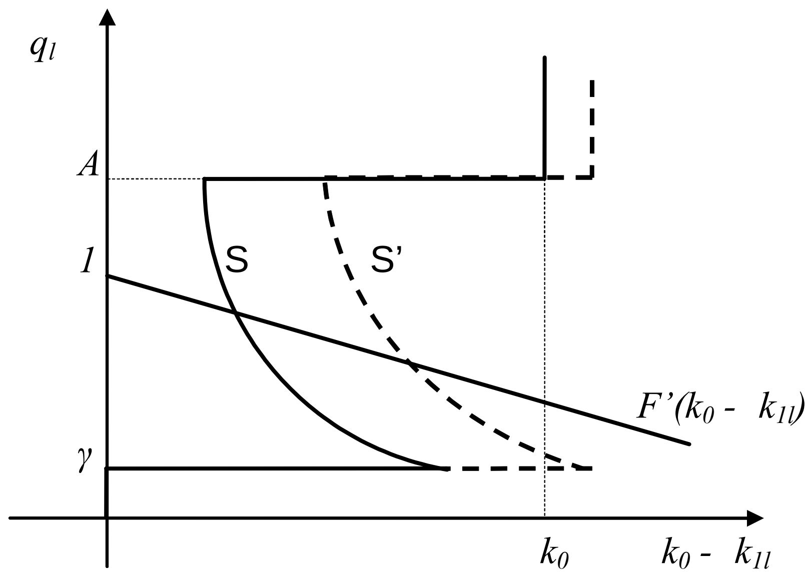


Figure 1: Asset market equilibrium

The equilibrium is determined at the point where the two curves meet.

Figure 1 can be used to show the relation between the financial contract and the asset price q_l. The choice of $\{b_{1s}\}$ affects the equilibrium price in two ways. An increase in either b_{1l} or b_{1h} increases the capital stock at date 0, given by

$$k_0 = \frac{1}{1 - \sum_s \pi_s b_{1s}} n,$$

and thus increases entrepreneurial losses in the low state. Moreover, an increase in b_{1l} directly increases repayments in the low state. Both channels lead to an increase in $-(x_l + \theta A - b_{1l}) k_0$ and to a fall in the equilibrium asset price. This mechanism is illustrated by the curve S' in Figure 1, which shows the effect of an increase in borrowing, leading to a rightward shift of the entrepreneurs' supply and to a lower equilibrium price.

3.3 Equilibrium hedging

Putting together entrepreneurs' optimality and the equilibrium determination of asset prices I can show that an equilibrium exists and I can characterize the equilibrium financial contract. From now on, I will use the superscript CE to denote equilibrium values.

Proposition 1 *There exists a unique symmetric competitive equilibrium. In equilibrium asset prices satisfy*

$$q_l^{CE} < q_h^{CE} = 1.$$

Depending on parameters, the equilibrium financial contract is of one of the following types:

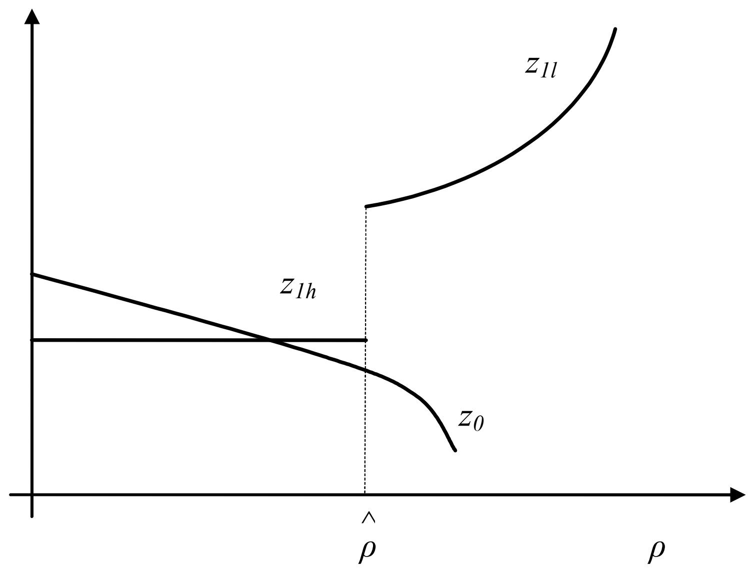
1. $0 \le b_{1h}^{CE} < \theta a_h + 1 - \gamma$ *and* $b_{1l}^{CE} = 0$;
2. $b_{1h}^{CE} = \theta a_h + 1 - \gamma$ *and* $b_{1l}^{CE} = 0$;
3. $b_{1h}^{CE} = \theta a_h + 1 - \gamma$ *and* $0 \le b_{1l}^{CE} \le \theta a_l + q_l^{CE} - \gamma$.

This proposition shows that there is a "pecking order" of repayments in different aggregate states. Entrepreneurs must first exhaust their borrowing capacity in the high state (setting $b_{1h} = \theta a_h + 1 - \gamma$), before they start borrowing against revenue in the low state (setting $b_{1l} > 0$). In the low state, the entrepreneurs are poor and the demand for assets is low. The associated fall in asset prices increases z_{1l} and induces entrepreneurs, ex ante, to reduce their promised repayments in that state. In equilibria of types 1 and 2, this incentive is sufficiently strong that entrepreneurs keep their promised repayments to zero in the low state. In equilibria of type 3, instead, the benefits from hedging are dominated by the return on investment at date 0. In this case, entrepreneurs decide to offer positive repayments also in the low state, in order to raise more capital at date 0.

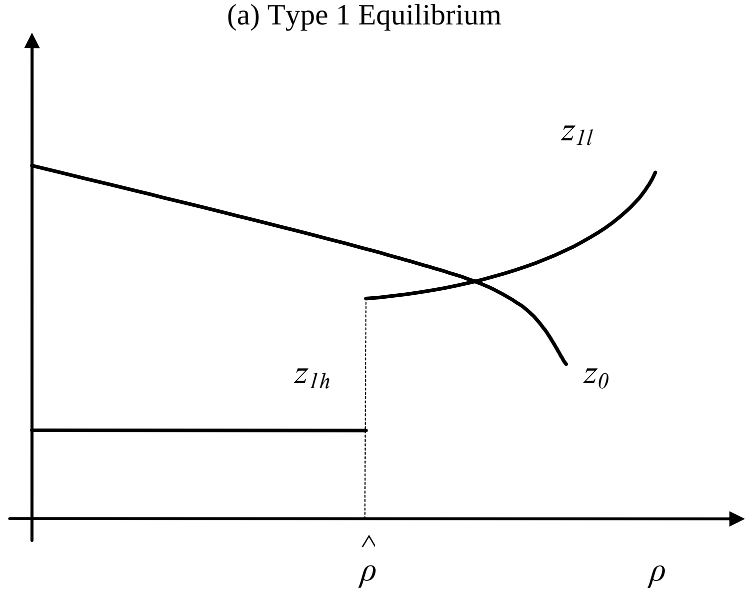
The general principle behind this result is that endogenous movements in asset prices determine the entrepreneurs' incentive to hedge aggregate shocks. In Section 4, I will show that the social benefits of this hedging are, in general, different from the private benefits.

3.4 A graphical illustration

An implication of Proposition 1 is that the equilibrium financial contract can be summarized by the variable $\rho \equiv \sum_s \pi_s b_{1s}$ capturing the ratio of outside borrowing to total capital invested at date 0. For low levels of ρ all the borrowing is against revenue in the high state, while if ρ is greater than the cutoff $\hat{\rho} \equiv \pi_h (\theta a_h + 1 - \gamma)$ the entrepreneurs also borrow against revenue in the low state. To illustrate the determination of the equilibrium financial contract, in Figure 2, I plot the relation between ρ and the rates of return on entrepreneurial wealth z_0 and z_{1s}. For each value of ρ, I derive the corresponding values of the state-contingent payments $\{b_{1s}\}$ and the equilibrium asset prices, proceeding as in Section 3.2. Given these asset prices, I derive



(a) Type 1 Equilibrium



(b) Type 3 Equilibrium

Figure 2: The borrowing ratio ρ and the rates of return on entrepreneurial wealth

the corresponding values of z_0 and z_{1s}, as in 3.1, and look for an optimal financial contract. Notice that when $\rho < \hat{\rho}$ the entrepreneur is choosing an interior solution for b_{1h}. In this case, the relevant marginal trade-off is between investing more at date 0 and investing in the high state at date 1. On the other hand, when $\rho > \hat{\rho}$ the trade-off is between investing at date 0 and investing in the low state at date 1. Hence, in the first region I plot z_{1h} as the relevant ex post rate of return, while in the second region I plot z_{1l}. The ex ante return is always equal to z_0.

Consider now how an increase in borrowing changes the returns to entrepreneurial wealth in periods 0 and 1. As ρ increases the price of capital q_l falls. This tends to reduce the ex ante return on entrepreneurial wealth, z_0, given that entrepreneurs face bigger expected capital losses in period 1.[15] At the same time, the ex post return on entrepreneurial wealth tends to increase for two reasons. First, if ρ crosses the cutoff $\hat{\rho}$ there is a discrete upward jump in the rate of return, since $z_{1l} > z_{1h}$. Furthermore, once above $\hat{\rho}$, the rate of return z_{1l} keeps increasing. As q_l falls an extra dollar available in the low state earns a higher return between periods 1 and 2. The equilibrium is determined at the point where the two rates of return are equalized, except in the cases where the entrepreneur is against a corner for both b_{1l} and b_{1h}. Panels (a) and (b) of Figure 2 illustrate two cases of interior equilibria. In the first case the equilibrium is of type 1 and $z_{1h} = z_0$, in the second the equilibrium is of type 3 and $z_{1l} = z_0$.

4 Welfare

Let me now turn to efficiency. Consider a planner who, at date 0, can choose the financial contract $\langle d_0, \{d_{1s}\}, \{d_{2s}\}\rangle$. The planner faces the same constraints as the private economy, in particular: (i) the financial contract is subject to default and renegotiation, and (ii) the allocation of used capital in period 1 is determined on an anonymous spot market. The only difference between the planner and the individual entrepreneur is that the planner takes into account the relation between the financial contract and the equilibrium price on the capital market. That is, instead of taking $\{q_s\}$ as given, the planner's problem includes the constraints

$$q_s = F'\left(k_{1s}^T\right), \tag{17}$$

$$k_{1s}^T = (k_0 - k_{1s})_+ . \tag{18}$$

[15] Notice that, in general, the relation between ρ and z_0 is not necessarily monotone, given that the expression (15) also includes z_{1l}. However, the difference $z_{1s} - z_0$ is locally monotone in ρ, around any equilibrium, which ensures that the equilibrium is unique. See the proof of Proposition 1 for the detailed derivations behind this statement and for analytical derivations which mirror the graphical presentation in this section.

As in the previous section, I will describe a financial contract in terms of the initial loan d_0 and the repayments per unit of capital $\{b_{1s}, b_{2s}\}$. The Pareto frontier is defined by the following planner's problem. Fix a given utility level for the consumers, $\bar{U}$. The planner chooses $\langle d_0, \{b_{1s}, b_{2s}\}\rangle$, k_0, and $\{k_{1s}\}$ to maximize the entrepreneurs' expected utility (7), subject to the budget constraints (8)-(9), the no-default constraints (10)-(11), the constraints (17)-(18), and a constraint on consumers' expected utility, which takes the place of the consumers' participation constraint,

$$3e - d_0 + \sum_s \pi_s b_{1s} k_0 + \sum_s \pi_s \left(F\left(k_{1s}^T\right) - q_s k_{1s}^T\right) \geq \bar{U}. \tag{19}$$

Since asset prices also determine the profits of the traditional sector, this is taken into account when writing constraint (19).

Let me first show that the only substantial difference between this problem and the problem of the individual entrepreneur, is, indeed, the endogeneity of asset prices. Suppose asset prices are set at their competitive equilibrium level, i.e., replace (17)-(18) with $q_s = q_s^{CE}$. Set also $\bar{U}$ at its equilibrium level, denoted by U^{CE}. Then it would be optimal for the planner to choose k_{1s}^T to maximize $F\left(k_{1s}^T\right) - q_s^{CE} k_{1s}^T$, so as to relax constraint (19), and thus choose $k_{1s}^T = k_{1s}^{T,CE}$. Since $U^{CE} = 3e + \sum_s \pi_s \left(F\left(k_{1s}^{T,CE}\right) - q_s^{CE} k_{1s}^{T,CE}\right)$ it follows that (19) could be replaced by (12). Since (17) and (18) are no longer present, the planner's problem would then be identical to the individual problem, and the competitive financial contract would be optimal.[16]

I now go back to the original formulation of the planner's problem. In order for that problem to be well defined, $\bar{U}$ cannot be too large, or the constraint set may be empty. Let me assume that this is the case, and let me also assume that $\bar{U}$ is such that the entrepreneur gets positive utility. Moreover, as in the previous section, I will ignore the non-negativity constraints for consumers' consumption, assuming that e is sufficiently large.

The following proposition gives a characterization of a constrained efficient allocation, which is denoted by an asterisk.

Proposition 2 *Suppose the value of $\bar{U}$ is such that the entrepreneur can achieve positive utility. Then, a socially optimal allocation satisfies the following conditions. Asset prices satisfy*

$$q_l^* < q_h^* = 1,$$

[16] Kehoe and Levine (1993) call this property "conditional constrained efficiency."

and promised repayments satisfy

$$\begin{array}{rcll} b_{1s}^* & = & 0 & \textit{if } \lambda^* < z_{1s}^*, \\ b_{1s}^* & \in & [0, \theta a_s + q_s^* - \gamma] & \textit{if } \lambda^* = z_{1s}^*, \\ b_{1s}^* & = & \theta a_s + q_s^* - \gamma & \textit{if } \lambda^* > z_{1s}^*, \end{array}$$

$$b_{2s}^* = \theta A,$$

for $s = l, h$, where λ^* is the Lagrange multiplier on constraint (19) and

$$z_{1s}^* = \frac{(1-\theta) A}{q_s^* - \theta A}. \tag{21}$$

The value of λ^* satisfies

$$\lambda^* \leq z_0^* = \frac{\sum_s \pi_s z_{1s}^* \left(q_s^* + x_s - b_{1s}^*\right)}{1 - \sum_s \pi_s b_{1s}^*}, \tag{22}$$

which holds as a strict inequality if $z_0^* \neq z_{1l}^*$.

The characterization of the financial contract parallels the result in Lemma 2. As in the individual problem the planner chooses the promised repayments in each state comparing the marginal return on entrepreneurial wealth in periods 0 and 1. The social return on entrepreneurial wealth in period 1, z_{1s}^*, is identical to the private rate of return, as can be seen comparing (14) and (21). However, the social rate of return in period 0 is now captured by λ^*. Inequality (22) shows that λ^* is smaller or equal than the corresponding expression for the private rate of return.

4.1 Over-borrowing

Let me now use this characterization to show that over-borrowing can arise in equilibrium. Recall that $\rho = \sum_s \pi_s b_{1s}$ is the ratio between the net present value of promised repayments and capital invested at date 0, introduced in Section 3.4. Thanks to Proposition 2, a socially optimal financial contract can be fully characterized by the ratio ρ, as it was the case for the competitive equilibrium contract. Therefore, I can focus on the comparison of ρ^* and ρ^{CE}. To prove next proposition it is convenient to introduce a slightly stronger version of Assumption C.[17]

[17] Assumption C' is stronger because $\eta > 1$. Notice that $\lim_{\pi_l \to 0} \eta = 1$, so the two assumptions tend to be equivalent for low values of π_l.

Assumption C' *The function F satisfies the following condition*

$$F'\left(k^T\right) - \theta A + \eta F''\left(k^T\right) k^T > 0,$$

where $\eta \equiv \left(1 - \pi_h\left(\theta a_h + 1 - \gamma\right) - \pi_l\left(x_l + \theta A\right)\right) / \left(1 - \sum_s \pi_s\left(\theta a_s + 1 - \gamma\right)\right)$.

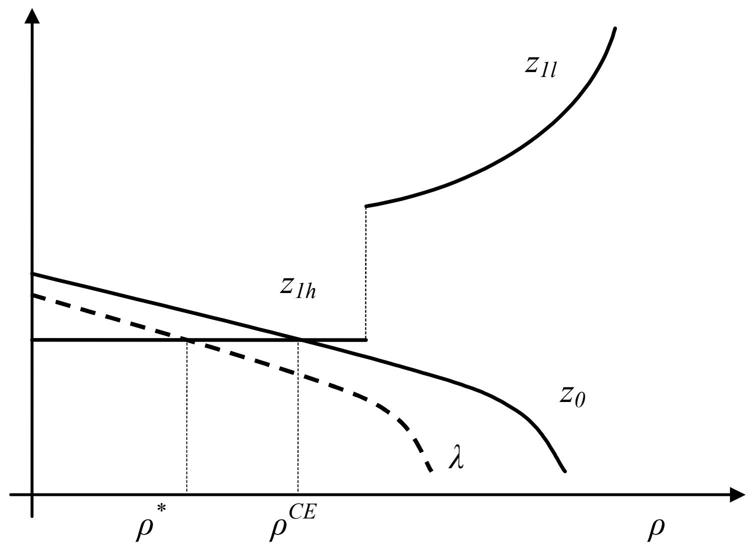
This condition is sufficient to show that when the planner increases the borrowing ratio ρ, this increases the sales of used capital by entrepreneurs in the low state. Under Assumptions A, B, and C', the following proposition shows that under-borrowing never arises in equilibrium, and over-borrowing arises if the equilibrium is of type 1.

Proposition 3 *(over-borrowing) Let $\bar{U} = U^{CE}$, then a constrained efficient financial contract satisfies $\rho^* \leq \rho^{CE}$. The inequality is strict if the equilibrium is of type 1.*

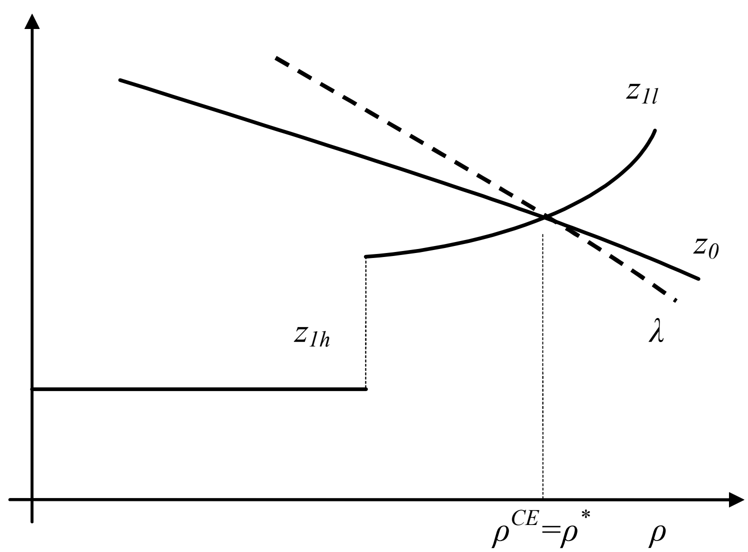
This result is due to the presence of a pecuniary externality: the financial decisions of the entrepreneurs affect the equilibrium price on the capital market in period 1, and this price affects the allocation of wealth between entrepreneurs and consumers. To clarify why this leads to a welfare loss consider the following experiment. Suppose the equilibrium is of type 1 and suppose entrepreneurs and consumers get together in period 0 and coordinate to reduce entrepreneurial investment in the initial period by $dk_0 < 0$, by reducing b_{1h}. Since $z_0 = z_{1h}$, the direct effect of this change on the entrepreneurs' utility is zero. However, in general equilibrium this change implies a reduced supply of used capital and a higher asset price in the low state, $dq_l > 0$. Since entrepreneurs are sellers of capital in that state, an increase in the asset price increases entrepreneurial wealth by $\left(k_0 - k_{1l}\right) dq_l$ dollars. At the same time, by the envelope theorem, $\left(k_0 - k_{1l}\right) dq_l$ corresponds to the reduction in profits for firms in the traditional sector. Suppose the entrepreneurs compensate the consumers for this profit loss by giving them $\pi_l\left(k_0 - k_{1l}\right) dq_l$ at date 0. The marginal cost of this transfer is $z_0 \pi_l\left(k_0 - k_{1l}\right) dq_l$ since z_0 is the entrepreneurs' marginal utility of funds. The expected marginal benefit associated to the increase in asset prices at date 1 is $\pi_l z_{1l}\left(k_0 - k_{1l}\right) dq_l$. The net effect of this local perturbation on the entrepreneurs' utility is

$$\pi_l\left(z_{1l} - z_0\right)\left(k_0 - k_{1l}\right) dq_l, \tag{23}$$

which is positive since $z_{1l} > z_{1h} = z_0$ and $dq_l > 0$. This gives a Pareto improvement, as consumers are indifferent and entrepreneurs are better off.



(a) Type 1 Equilibrium



(b) Type 3 Equilibrium

Figure 3: The borrowing ratio ρ and the private and social returns to entrepreneurial investment.

4.2 A graphical illustration

Figure 3 is the analogous to Figure 2 for the case of the planner. Proposition 2 shows that the socially optimal contract satisfies the same pecking order identified for equilibrium contracts. Therefore, also in this case it is possible to represent the optimal financial contract in terms of the borrowing ratio ρ. For each level of ρ, I plot the corresponding values of z_{1s} and z_0, capturing the private benefits from ex ante and ex post investment. As in Figure 2, I plot z_{1h} when $\rho < \hat{\rho}$ and z_{1l} when $\rho > \hat{\rho}$. In the same picture I use a dashed line to plot λ, the social benefits on entrepreneurial investment at date 0. As shown in Proposition 2 the social benefits from investment at date 1 coincide with the private benefits and are captured by z_{1s}.

The difference between the private and social benefits of ex ante investment, z_0 and λ, are due to the pecuniary externality discussed above. An increase in borrowing has the effect of reducing q_l, and thus reallocating funds from entrepreneurs to consumers at date 1. The welfare effect of this reallocation is given by (23). Therefore, the difference between λ and z_0 has the same sign as $z_0 - z_{1l}$. In panel (a) the graph of λ is below that of z_0, given that z_{1l} is above z_0 for the whole range of ρ. In panel (b), instead, the graph of λ crosses that of z_0 at the point where $z_{1l} - z_0 = 0$. In the case depicted in panel (a), this implies that, at the competitive equilibrium $\lambda < z_{1h}$. Therefore, a reduction in borrowing leads to a Pareto improvement. In the case depicted in panel (b), instead, the competitive equilibrium arises precisely when $z_{1l} = z_0$, so there is no room for a Pareto improvement.

The latter argument suggests that equilibria of type 3 are constrained efficient. This is indeed the case in specific examples, such as the one depicted in panel (b). However, due to the non-concavity of the planner's problem, the result cannot be established in general.

4.3 Sources of inefficiency

The two imperfections introduced in the model are limited commitment on the entrepreneurs' and on the consumers' side. Both are at the roots of the inefficiency result. If I removed the entrepreneurs' commitment problem, by setting $\theta = 1$, the economy would reach a first-best allocation where all the consumption goods in periods 0 and 1 are devoted to investment.[18] In this case, not surprisingly, there is no inefficiency. Limited commitment on the consumers' side plays a subtler role. Consider the case of an equilibrium of type 1 where expression (23)

[18]Note that, in this case, the non-negativity constraint for consumers' consumption are binding. See the appendix for the formal analysis of this case.

is positive since $z_{1l} > z_0$. In this case, the reduction in borrowing is beneficial because the ex post value of funds to the entrepreneurs is larger than their value ex ante. This can only happen if the entrepreneur has limited ability to transfer resources between period 0 and period 1 (in state l). In the model, this happens because the constraint $b_{1l} \geq 0$ is binding, so the entrepreneurs are not allowed to buy more insurance against the low state. A coordinated reduction in borrowing, as described in 4.1, allows the entrepreneurs to partially circumvent the problem. Borrowing less ex ante leads to an increase in asset prices in the low state and, thus, it is an indirect way for entrepreneurs to transfer resources to the low state. This argument suggests that the pecuniary externality identified here will also be relevant in other environments where the entrepreneurs have difficulty channeling resources to the low state, for example, in models with full commitment on the consumers' side but without state-contingent debt.

It is useful to remark that in this framework the inefficiency is not due to fact that the price q_l affects the collateral available to entrepreneurs. As just noticed, the pecuniary externality identified matters when the constraint $b_{1l} \geq 0$ is binding. In that case, the collateral constraint $b_{1l} \leq \theta a_l + q_l - \gamma$ is slack and the fact that an increase in q_l increases the entrepreneur's borrowing capacity at date 0 is irrelevant for entrepreneurs. Asset prices matter here because they determine the asset side of entrepreneurs' balance sheets, not because of their effects on their capacity to borrow. In a model with endogenous asset prices in period 2, it would be possible to study the effect of asset prices on borrowing capacity in period 1, and to study a "collateral channel" different from the asset price channel discussed here.

As a final observation, notice that constraint (19), with $\bar{U} = U^{CE}$, ensures that consumers are as well off at a constrained efficient allocation as at the competitive allocation. One may ask, though, whether there should be an additional constraint to ensure that consumers participate voluntarily to the financial contract $\langle d_0^*, \{b_{1s}^*, b_{2s}^*\}\rangle$, that is, a constraint of the form $d_0^* \leq \sum \pi_s b_{1s}^*$. In the analysis so far, I have left this constraint aside, to simplify the exposition, but it can be shown that the constraint is not binding at a constrained efficient allocation. The proof of this claim is in the appendix (Lemma 8).

4.4 Remarks on policy

Regulatory interventions that impose minimum capitalization on financial firms are widespread in industrialized economies, and often their introduction is justified based on the idea that excessive leverage in the financial sector may bring about an increase in "systemic risk." The model presented here gives a welfare-based rationale to this idea. When the equilibrium is inefficient, policies that restricts borrowing ex ante can restore constrained efficiency. For example, the planner can impose a capital requirement of the form $\nu k_0 \leq n$, which imposes that a minimum fraction ν of the firm's assets are financed with insiders' capital. Given that both the planner's optimum and the entrepreneur's optimum follow the same "pecking order," a restriction of this type is sufficient to ensure that the efficient financial contract $\{b_{1s}^*\}$ is chosen.

Proposition 4 *Given a constrained efficient allocation, there is a capital requirement ν and a transfer τ_0 from entrepreneurs to consumers, such that the corresponding equilibrium is constrained efficient.*

An open question is how capital requirements should be calibrated for investments with different risky profiles. Existing capital requirements are usually based on the riskiness of the individual investment, using some measure of "value at risk." The framework of this paper can be extended to analyze models with different types of investment. In particular, one can consider a model where the cash-flows x_s of different investments have different exposure to the aggregate shock. In that case, the investments with a larger pecuniary externality are those which are more correlated with the aggregate shock, since they are the ones that contribute more to a drop in asset prices in the event of a bad shock. This points to the idea of optimal capital requirements that depend on macroeconomic correlations and not just on measures of individual risk. In particular, it might be desirable that investments with higher correlation with macroeconomic conditions be subject to tighter requirements.[19]

5 Conclusions

The policy debate on financial supervision and regulation has been recently shifting towards a "macroprudential" approach (Borio, 2003). According to this approach the regulator should be concerned most of all about the aggregate consequences of financial instability and the main source of instability is identified in the common exposure to macroeconomic risks across financial institutions. The present paper provides at the same time a warning and a justification for this approach. The warning is that aggregate volatility and some degree of financial fragility

[19] See Borio (2003, p.10) for a discussion of recent policy proposals that go in this direction.

are unavoidable in presence of financial constraints, and that a reduction in financial fragility can only be achieved at the cost of reducing investment ex ante. Defining the objective of the regulator only in terms of reducing volatility, and disregarding the productive effects of capital accumulation, may be misleading. On the other hand, the welfare analysis in this paper provides a justification for a macroprudential approach. In a framework with financial constraints, private agents may underestimate the damage associated to a contraction in their wealth and, therefore, policies that limit their losses in a crisis may be welfare improving.

In practice, capital requirements are imposed on a specific class of firms, typically on commercial banks and financial intermediaries. To have a fully fledged theory of capital requirements would require an explicit model of financial intermediation. If entrepreneurial firms, which are more financially constrained, are also more reliant on bank credit, capital requirements on banks can help to stabilize the balance sheet of the firms that need it most. The analysis of capital requirements in an explicit framework with intermediation remains a topic for future research.

The model presented shows that over-borrowing is a possibility. However, it also shows that in some equilibria (e.g., the equilibrium illustrated in panel (b) of Figure 3) the gains from ex ante investment are sufficiently large that both the private economy and the planner choose the same high level of borrowing. Therefore, the presence and the severity of over-borrowing in specific episodes becomes an empirical issue. In particular, the model indicates that some relevant quantitative questions that should be addressed are: how much the presence of financial distress contributes to the fall in asset prices during financial crises, and how much that contributes to the propagation of financial distress across the economy.

Let me conclude with some remarks on the notion of constrained efficiency used in this paper. The social planner introduced here is constrained to take as given both the limited commitment problem in financial contracts, and the fact that asset prices are determined in a spot market. The point of this exercise is both theoretical and practical. First, it is useful to consider a restrictive notion of constrained efficiency to identify minimal conditions under which a planner can improve upon the competitive allocation. Second, it is interesting to focus on policy interventions that impose restrictions on financial contracts, since they seem close to regulatory policies already in place. However, this environment naturally suggests that interventions in different markets (e.g., the asset market) can have important interactions with the equilibrium financial structure. Extensions of the welfare analysis to the case where the planner can intervene directly in the asset market are left to future research. This type of extensions will be of particular interest when one turns to monetary versions of the model and studies its implications for optimal monetary policy.[20]

Finally, one could allow the planner to directly intervene to relax the limited commitment constraints. In particular, Holmstrom and Tirole (1998) argue that the supply of public liquidity can alleviate the lack of commitment on the consumers' side. The government can issue state contingent bonds in period 0, and tax consumers in period 1 to repay these bonds. Let the tax on consumers be denoted by τ_s. Then, a model with public liquidity is formally equivalent to the model presented here, if the consumers' no-default constraint in period 1 is replaced by $d_{1s} + d_{2s} \geq -\tau_s$. This policy allows entrepreneurs to buy more insurance against the low state, by holding state-contingent government bonds. By setting a sufficiently high value for τ_s, the government is able to replicate the equilibrium of an economy with full commitment on the consumers' side. As argued above, in 4.3, this would eliminate the pecuniary externality identified in this paper. Clearly, there are a number of reasons why liquidity creation by the government may be costly and imperfect (e.g., the distortionary effects of taxation). In all these cases, the possibility of excessive borrowing remains open.

[20]Recent papers on optimal monetary policy in economies with financial frictions include Iacoviello (2005) and Faia and Monacelli (2007).

6 Appendix

6.1 Proof of Lemma 1

The consumer chooses $k_{1s}^T \geq 0$ to maximize expected utility, given by (1), that is, to maximize $F\left(k_{1s}^T\right) - q_s k_{1s}^T$. Recall that F is strictly concave and $F'(0) = 1$. Therefore, if $q_s \geq 1$ optimal investment is $k_{1s}^T = 0$, while if $q_s < 1$, k_{1s}^T is positive and satisfies the first order condition $F'\left(k_{1s}^T\right) = q_s$.

Recall that consumption goods can be turned into capital goods one for one but not the converse. This has two implications. First, by arbitrage, the price of capital must satisfy $q_s \leq 1$. Second, aggregate investment must be non-negative,

$$k_{1s}^T + k_{1s} - \chi_s k_0 \geq 0.$$

If aggregate investment is positive, then, by arbitrage, the price of capital must be $q_s = 1$. In this case, optimality for the traditional sector gives $k_{1s}^T = 0$, and, thus, investment by entrepreneurs must be positive, $k_{1s} - \chi_s k_0 > 0$. If, instead, aggregate investment is zero, then we have $k_{1s}^T = \chi_s k_0 - k_{1s}$. In this case, optimality for the traditional sector implies that either $k_{1s}^T = 0$ and $q_s = 1$ or $k_{1s}^T > 0$ and $q_s < 1$. The following conditions hold in all the cases considered

$$q_s = F'\left(k_{1s}^T\right), \tag{24}$$

$$k_{1s}^T = \left(\chi_s k_0 - k_{1s}\right)_+. \tag{25}$$

Recall that $\underline{q}$ is the lower bound for $F'\left(k_{1s}^T\right)$ and, due to Assumption B, $\underline{q} - \gamma$ is positive. These two facts imply that $q_s \geq \underline{q} > \gamma$. If any entrepreneur is scrapping capital in period 1, it is always a profitable deviation to pay the maintenance cost γ and sell the capital. Therefore, no scrapping takes place in equilibrium, $\chi_s = 1$. Substituting $\chi_s = 1$ in (25) completes the proof. ■

6.2 Lemma 3

The following is a useful additional result which will be used in the equilibrium characterization.

Lemma 3 *The equilibrium price of capital satisfies*

$$q_s - \theta A > 0, \tag{26}$$

for each s.

Proof. If $k_{1s}^T = 0$ then $q_s = 1$ and the result follows immediately from Assumption A. If instead $k_{1s}^T > 0$, Assumption C implies that $\left(F'\left(k_{1s}^T\right) - \theta A\right) k_{1s}^T > \left(F'(0) - \theta A\right) 0 = 0$. Since $q_s = F'\left(k_{1s}^T\right)$ inequality (26) follows. ■

6.3 Proof of Lemma 2

Consider the problem of maximizing (7) subject to (8)-(12) and non-negativity constraints for k_0 and $\{k_{1s}\}$. Given the bounds for equilibrium prices (6) and (26), and given Assumption A, it is possible to show that the constraint set is non-empty and compact, hence a solution exists.

Consider the entrepreneur's problem defined in terms of the original variables $\{d_{1s}, d_{2s}\}$. It can be shown that the problem stated above (in terms of $\{b_{1s}, b_{2s}\}$), is equivalent to the original problem. That is, for any $\langle d_0, \{d_{1s}, d_{2s}\}\rangle$ and $\langle k_0, \{k_{1s}\}\rangle$ in the feasible set of the original problem, there is a $\langle d_0, \{b_{1s}, b_{2s}\}\rangle$ and $\langle k_0, \{k_{1s}\}\rangle$ in the feasible set of the transformed problem, which achieves the same payoff; the converse is also true. These statements are obvious when $k_0 > 0$ and $k_{1s} > 0$ for all s. When $k_0 = 0$ or $k_{1s} = 0$ for some s, they rely on the following facts: (i) $k_0 = 0$, (2), and (4) imply $d_{1s} + d_{2s} = 0$; (ii) $k_{1s} = 0$, (3), and (5) imply $d_{2s} = 0$.

Let me replace (8) and (12) with the constraint

$$k_0 \leq n + \sum_s \pi_s b_{1s} k_0. \tag{8'}$$

It is easy to show that if (8') is satisfied, there exists a d_0 such that both (8) and (12) are satisfied.

Let z_0 and $\pi_s z_{1s}$ denote the Lagrange multipliers associated, respectively, to (8') and (9). An optimum is characterized by the following first order conditions

$$\begin{aligned} -z_0 + z_0 \sum_s \pi_s b_{1s} + \sum_s \pi_s z_{1s} (q_s + x_s - b_{1s}) &\leq 0, & (27)\\ \pi_s (A - b_{2s}) - \pi_s z_{1s} (q_s - b_{2s}) &\leq 0, & (28) \end{aligned}$$

which must hold with strict equality if, respectively, $k_0 > 0$ or $k_{1s} > 0$,

$$\begin{aligned} &\text{if } \pi_s z_0 k_0 - \pi_s z_{1s} k_0 < 0 \text{ then } b_{1s} = 0, & (29a)\\ &\text{if } \pi_s z_0 k_0 - \pi_s z_{1s} k_0 = 0 \text{ then } b_{1s} \in [0, \theta a_s + q_s - \gamma], & (29b)\\ &\text{if } \pi_s z_0 k_0 - \pi_s z_{1s} k_0 > 0 \text{ then } b_{1s} = \theta a_s + q_s - \gamma, & (29c) \end{aligned}$$

$$\begin{aligned} &\text{if } \pi_s z_{1s} k_{1s} - \pi_s k_{1s} < 0 \text{ then } b_{2s} = 0, & (30a)\\ &\text{if } \pi_s z_{1s} k_{1s} - \pi_s k_{1s} = 0 \text{ then } b_{2s} \in [0, \theta A], & (30b)\\ &\text{if } \pi_s z_{1s} k_{1s} - \pi_s k_{1s} > 0 \text{ then } b_{2s} = \theta A. & (30c) \end{aligned}$$

Lemma 3 and no default imply that $q_s > \theta A \geq b_{2s}$. Condition (6) and $A > 1$ imply $A > q_s$. Then (28) gives

$$z_{1s} \geq \frac{A - b_{2s}}{q_s - b_{2s}} > 1,$$

for all s. Using condition (30c) I get $b_{2s} = \theta A$ for $s = l, h$. Moreover, since $z_{1s} > 0$ the constraint (9) is binding and

$$k_{1s} = \frac{q_s + x_s - b_{1s}}{q_s - \theta A} k_0. \tag{31}$$

Rearranging condition (27) I get

$$z_0 \geq \frac{\sum_s \pi_s z_{1s} (q_s + x_s - b_{1s})}{1 - \sum_s \pi_s b_{1s}},$$

where $z_{1s} > 0$, $q_s + x_s - b_{1s} > 0$ from (10) and $\theta < 1$, and $\sum_s \pi_s b_{1s} < 1$ from (10) and Assumption A. Therefore, $z_0 > 0$, which implies that constraint (8') is binding and $k_0 = n + \sum_s \pi_s b_{1s} k_0 \geq n > 0$. This implies that (27) holds as an equality, and gives (15). Then, (31) implies that $k_{1s} > 0$, which, in turn, shows that (28) holds as an equality, giving (14). Finally, conditions (29a)-(29c) give (13a)-(13c) in the lemma. ■

6.4 Proof of Proposition 1

The proof is split in three lemmas. I first prove the characterization part, then existence and uniqueness.

Lemma 4 *In any symmetric equilibrium $q_h = 1 > q_l$ and the financial contract is of one of the types 1 to 3 defined in Proposition 1.*

Proof. First, let me prove that the prices satisfy $q_h = 1 > q_l$. Rewrite the budget constraint (9) as

$$(q_s - \theta A)(k_{1s} - k_0) = (x_s + \theta A - b_{1s}) k_0.$$

Lemma 3 implies that $q_s - \theta A > 0$. Given that $k_0 > 0$, as shown in Lemma 2, to prove the statement regarding asset prices it is sufficient to prove that $x_h + \theta A - b_{1h} > 0$ and $x_l + \theta A - b_{1l} < 0$, so that entrepreneurs' investment is positive in h and negative in l. To prove the first inequality notice that the no-default constraint, $b_{1h} \leq \theta a_h + q_h - \gamma$, and inequality (6) imply

$$x_h + \theta A - b_{1h} \geq (1-\theta) a_h + \theta A - 1 > 0,$$

where the second inequality follows from Assumption B. To prove the second inequality notice that $x_l + \theta A - b_{1l} \leq x_l + \theta A < 0$ follows from the consumers' no-default constraint and Assumption B.

Having proved that $q_h = 1 > q_l$, it follows that

$$z_{1h} = \frac{(1-\theta) A}{1 - \theta A} < \frac{(1-\theta) A}{q_l - \theta A} = z_{1l}.$$

Therefore, one of the following three cases applies (1) $z_0 \leq z_{1h} < z_{1l}$, (2) $z_{1h} < z_0 < z_{1l}$, (3) $z_{1h} < z_{1l} \leq z_0$. Applying Lemma 2 these three cases give the equilibrium financial contracts of types 1 to 3. ■

This characterization implies that the equilibrium financial contract takes the form

$$b_{1l} = \frac{1}{\pi_l} \max\{\rho - \hat{\rho}, 0\}, \tag{32}$$

$$b_{1h} = \frac{1}{\pi_h} \min\{\rho, \hat{\rho}\}, \tag{33}$$

for some parameter $\rho \geq 0$, where $\hat{\rho} \equiv \pi_h (\theta a_h + 1 - \gamma)$. That is, the equilibrium financial contract is fully characterized by ρ. Notice that, by construction, $\rho = \sum_s \pi_s b_{1s}$, i.e. the parameter ρ captures the ratio of outside borrowing to total capital invested at date 0.

The next Lemma contains useful results on the relation between ρ and the equilibrium price q_l.

Lemma 5 *There is an upper bound $\bar{\rho}$ such that the contract $\{b_{1s}\}$ given by (32)-(33) is consistent with no default if and only if $\rho \in [0, \bar{\rho}]$. For each $\rho \in [0, \bar{\rho}]$ there is a unique equilibrium in the low state capital market. The associated equilibrium price is given by the function $q_l = Q(\rho)$, which is continuous and decreasing, and is differentiable except at $\rho = \hat{\rho}$.*

Proof. For any $\rho \geq 0$, let $\{b_{1s}\}$ be given by (32)-(33). To find the corresponding equilibrium in the l-state asset market I need to find a price q_l and quantities k_{1l}^T and k_{1l} such that $q_l = F'(k_{1l}^T)$, $(q_l - \theta A)(k_{1l} - k_0) = (x_l + \theta A - b_{1l}) k_0$ (from the entrepreneurs' budget constraint), and $k_{1l}^T = k_0 - k_{1l}$. To find the equilibrium, I define the function

$$H(k_{1l}^T; b_{1l}, b_{1h}) \equiv \left(F'(k_{1l}^T) - \theta A\right) k_{1l}^T + \left(x_l + \theta A - \tilde{b}_{1l}\right) k_0 \tag{34}$$

where

$$\tilde{b}_{1l} = \min\left\{b_{1l}, \theta a_l + F'(k_{1l}^T) - \gamma\right\}, \tag{35}$$

$$k_0 = \frac{1}{1 - \sum_s \pi_s b_s} n, \tag{36}$$

and look for a k_{1l}^T that solves $H(k_{1l}^T; b_{1l}, b_{1h}) = 0$. Notice that for every ρ, (33) ensures that $b_{1h} \leq \theta a_h + 1 - \gamma$, so the no-default condition is satisfied in state h. However, with no restrictions on ρ nothing ensures that no-default is satisfied in state l. For the moment I use condition (35) to ensure that the no-default condition is satisfied by $(\tilde{b}_{1l}, b_{1h})$. This construction will allow me, eventually, to find a $\bar{\rho}$ such that no-default is satisfied if and only if $\rho \leq \bar{\rho}$.

Step 1. First, I want to prove that for each pair $\{b_{1s}\}$ there exists a unique $k_{1l}^T \in [0, k_0]$ that solves $H\left(k_{1l}^T; b_{1l}, b_{1h}\right) = 0$. To do that, I will show that the function H: (i) is continuous and increasing in k_{1l}^T, (ii) is negative at $k_{1l}^T = 0$, and (iii) is positive at $k_{1l}^T = k_0$. Point (i) follows from Assumption C and the concavity of F, which implies that $\tilde{b}_{1l}$ is non-increasing in k_{1l}^T. Point (ii) follows since $\left(x_l + \theta A - \tilde{b}_{1l}\right) k_0 < 0$ (from $\tilde{b}_{1l} \geq 0$ and Assumption B). To prove (iii) notice that

$$H(k_0; b_{1l}, b_{1h}) = \left(a_l + F'(k_0) - \gamma - \tilde{b}_{1l}\right) k_0 \geq (1-\theta) a_l k_0 > 0,$$

where the first inequality follows from the definition of $\tilde{b}_{1l}$, the second from $a_l > 0$. The intermediate value theorem implies that there exists a k_{1l}^T which solves $H\left(k_{1l}^T; b_{1l}, b_{1h}\right) = 0$. Since H is monotone in k_{1l}^T, the solution is unique.

Step 2. Next, I define the function $Q(\rho)$ and show that it is continuous and decreasing. For any $\rho \geq 0$, let b_{1l} and b_{1h} be given by (32) and (33), let k_{1l}^T solve $H\left(k_{1l}^T; b_{1l}, b_{1h}\right) = 0$ and let $Q(\rho) = F'\left(k_{1l}^T\right)$. Continuity can be easily established. To prove that Q is decreasing, note that $x_l + \theta A - \tilde{b}_{1l}$ is negative, k_0 is increasing in both b_{1l} and b_{1h}, and $x_l + \theta A - \tilde{b}_{1l}$ is non-increasing in b_{1l}. Therefore, H is decreasing in both b_{1l} and b_{1h}. Moreover, H is increasing in k_{1l}^T. This implies that the k_{1l}^T which solves $H\left(k_{1l}^T; b_{1l}, b_{1h}\right) = 0$ is increasing in ρ, since, if ρ increases either b_{1l} or b_{1h} must increase. The concavity of F then implies that $Q(\rho)$ is decreasing.

Step 3. Finally, I find the upper bound $\bar{\rho}$ and argue that the function $Q(\rho)$ is differentiable, except at $\rho = \hat{\rho}$. Let $\bar{\rho}$ be such that

$$\frac{1}{\pi_l}(\bar{\rho} - \hat{\rho}) = \theta a_l + Q(\bar{\rho}) - \gamma.$$

This equation admits a solution $\bar{\rho} \in [\hat{\rho}, \hat{\rho} + \pi_l(\theta a_l + 1 - \gamma)]$ by the intermediate value theorem, given that $0 < \theta a_l + Q(\hat{\rho}) - \gamma$ and $\theta a_l + 1 - \gamma \geq \theta a_l + Q(\hat{\rho} + \pi_l(\theta a_l + 1 - \gamma)) - \gamma$ (by (6) and Assumption B). The solution is unique because Q is decreasing. Moreover, again given that Q is decreasing, I have $b_{1l} = \frac{1}{\pi_l}(\rho - \hat{\rho}) \leq \theta a_l + Q(\rho) - \gamma$ if and only if $\rho \leq \bar{\rho}$. Restricting the function Q to $[0, \bar{\rho}]$, I can replace $\tilde{b}_{1l}$ with b_{1l} in (34), and apply the implicit function theorem to show that Q is differentiable for $\rho \neq \hat{\rho}$. ■

I can now prove existence and uniqueness.

Lemma 6 *There exists a unique symmetric competitive equilibrium.*

Proof. *Step 1.* First, I will define a function $\zeta : [0, \bar{\rho}] \to \mathbb{R}$. For each ρ, let b_{1l} and b_{2l} be given by (32) and (33), let $q_h = 1$ and $q_l = Q(\rho)$. Substitute in (14) and (15), to obtain $\{z_{1s}\}$ and z_0, and let

$$\zeta(\rho) \equiv \begin{cases} z_{1h} - z_0 & \text{if } \rho \in [0, \hat{\rho}] \\ z_{1l} - z_0 & \text{if } \rho \in (\hat{\rho}, \bar{\rho}] \end{cases}.$$

The function ζ is continuous and differentiable except at $\hat{\rho}$. Note that ζ corresponds to the difference between the z_{1s} and z_0 plotted in Figure 2.

Step 2. The function ζ satisfies two properties. First, it satisfies $\zeta(\hat{\rho}) < \lim_{\rho \to \hat{\rho}^+} \zeta(\rho)$. This follows from the inequality $z_{1h} < z_{1l}$, which can be proved for any $\rho \in [0, \bar{\rho}]$ proceeding as in the proof of Lemma 4. Second, if $\zeta(\rho) = 0$ and ζ is differentiable at ρ, then $\zeta'(\rho) > 0$. To prove this claim, consider first the case $\rho < \hat{\rho}$. In this case some algebra shows that

$$\zeta'(\rho) = \frac{1}{1 - \sum \pi_s b_{1s}} \left(\pi_l z_{1l} \frac{x_l + \theta A - b_{1l}}{q_l - \theta A} Q'(\rho) + \zeta(\rho) \right).$$

The last expression is positive given that $x_l + \theta A - b_{1l} < 0$ (see the proof of Lemma 4), $Q'(\rho) < 0$ (from Lemma 5), and $\zeta(\rho) = 0$. If $\rho > \hat{\rho}$ then

$$\zeta'(\rho) = -\frac{z_{1l}}{q_l - \theta A} Q'(\rho) + \frac{1}{1 - \sum_s \pi_s b_{1s}} \left(\pi_l z_{1l} \frac{x_l + \theta A - b_{1l}}{q_l - \theta A} Q'(\rho) + \zeta(\rho) \right),$$

which is also positive.

Step 3. Summarizing the properties derived in step 2: ζ is continuous except at $\hat{\rho}$, where it has an upward jump, and, if it crosses the horizontal axis, it is locally increasing at the point of crossing. These properties imply that there exists one and only one $\rho^{CE} \in [0, \bar{\rho}]$ which satisfies one of the following conditions: (i) $\rho^{CE} = 0$ and $\zeta(0) \geq 0$; (ii) $\rho^{CE} \in (0, \hat{\rho})$ and $\zeta\left(\rho^{CE}\right) = 0$; (iii) $\rho^{CE} = \hat{\rho}$ and $\zeta(\hat{\rho}) \leq 0 \leq \lim_{\rho \to \hat{\rho}^+} \zeta(\rho)$; (iv) $\rho^{CE} \in (\hat{\rho}, \bar{\rho})$ and $\zeta\left(\rho^{CE}\right) = 0$; (v) $\rho^{CE} = \bar{\rho}$ and $\zeta(\bar{\rho}) \leq 0$. For each of these cases, it is possible to construct a competitive equilibrium. For example, case (ii) gives $b_{1l}^{CE} = 0$, $b_{1h}^{CE} = \rho^{CE}/\pi_h$ and $z_{1l}^{CE} > z_{1h}^{CE} = z_0^{CE}$, showing that the entrepreneur's optimality conditions derived in Lemma 2 are satisfied. All remaining equilibrium conditions are satisfied by construction.

Step 4. To prove uniqueness notice that the characterization in Lemma 4 implies that if $\rho^{CE} = \sum \pi_s b_{1s}^{CE}$ and $\left\{b_{1s}^{CE}\right\}$ is the equilibrium contract, then ρ^{CE} must satisfy one of the conditions (i)-(v) described in step 3. ■

The following is a corollary of Lemma 6, which will be useful in the welfare analysis. It follows immediately from step 3 of the proof.

Corollary 1 *Let ζ be the function defined in the proof of Lemma 6 and let ρ^{CE} be the equilibrium level of ρ. Then, $\zeta(\rho) > 0$ for all $\rho > \rho^{CE}$.*

6.5 Proof of Proposition 2

Define the function G

$$G(y) = \begin{cases} F(y) & \text{if } y \geq 0 \\ y & \text{if } y < 0 \end{cases}.$$

Since $F'(0) = 1$ this function is differentiable. The planner's problem can then be written as follows

$$\max_{k_0, \{k_{1s}, b_{1s}, b_{2s}\}_{s=l,h}} \sum_s \pi_s (A - b_{2s}) k_{1s}$$

subject to

$$\begin{aligned}
& n + 3e - \bar{U} - k_0 + \sum_s \pi_s b_{1s} k_0 + \sum_s \pi_s \left(G(k_0 - k_{1s}) - G'(k_0 - k_{1s})(k_0 - k_{1s})\right) \geq 0, && (\lambda) \\
& G'(k_0 - k_{1s})(k_0 - k_{1s}) + (x_s - b_{1s}) k_0 + b_{2s} k_{1s} \geq 0, && (\pi_s \mu_s) \\
& 0 \leq b_{1s} \leq \theta a_s + G'(k_0 - k_{1s}) - \gamma, && (\pi_s \nu_s) \\
& 0 \leq b_{2s} \leq \theta A, \\
& k_{1s}, k_0 \geq 0.
\end{aligned}$$

In parentheses, I report the Lagrange multipliers corresponding to the first three sets of constraints. The multiplier ν_s refers to the inequality $b_{1s} \leq \theta a_s + G'(k_0 - k_{1s}) - \gamma$. I will write $G_s'^*$ for $G'(k_0^* - k_{1s}^*)$ and $G_s''^*$ for $G''(k_0^* - k_{1s}^*)$. As in the case of the entrepreneur's problem, the planner's problem can be stated in terms of the variables $\{b_{1s}\}$ and $\{b_{2s}\}$, even if k_0 or $\{k_{1s}\}$ are zero (see the proof of Lemma 2). The rest of the proof is split in several steps.

Step 1. I derive the Kuhn-Tucker necessary first order conditions for an optimum. First, the optimality conditions for k_0 and k_{1s},

$$\lambda\left(\sum_s \pi_s\left[b_{1s}^* - G_s''^*\left(k_0^* - k_{1s}^*\right)\right] - 1\right) + \sum_s \pi_s\mu_s\left[x_s + G_s'^* - b_{1s}^* + G_s''^*\left(k_0^* - k_{1s}^*\right)\right] + \sum_s \pi_s\nu_s G_s''^* \le 0, \tag{37}$$

and

$$\pi_s\left(A - b_{2s}^*\right) + \lambda\pi_s G_s''^*\left(k_0^* - k_{1s}^*\right) + \pi_s\mu_s\left(b_{2s}^* - G_s'^* - G_s''^*\left(k_0^* - k_{1s}^*\right)\right) - \pi_s\nu_s G_s''^* \le 0, \tag{38}$$

which must hold with strict equality if, respectively, $k_0 > 0$ or $k_{1s} > 0$. The conditions for b_{1s} and b_{2s} give

$$\begin{aligned}
&\text{if } \pi_s\lambda k_0^* - \pi_s\mu_s k_0^* &<& \ 0 \text{ then } \nu_s = 0 \text{ and } b_{1s}^* = 0, & \text{(39a)}\\
&\text{if } \pi_s\lambda k_0^* - \pi_s\mu_s k_0^* &=& \ 0 \text{ then } \nu_s = 0 \text{ and } b_{1s}^* \in \left[0, \theta a_s + G_s'^* - \gamma\right], & \text{(39b)}\\
&\text{if } \pi_s\lambda k_0^* - \pi_s\mu_s k_0^* &>& \ 0 \text{ then } \nu_s = \left(\lambda - \mu_s\right)k_0^* \text{ and } b_{1s}^* = \theta a_s + G_s'^* - \gamma, & \text{(39c)}
\end{aligned}$$

$$\begin{aligned}
&\text{if } \pi_s\mu_s k_{1s}^* - \pi_s k_{1s}^* &<& \ 0 \text{ then } b_{2s}^* = 0, & \text{(40a)}\\
&\text{if } \pi_s\mu_s k_{1s}^* - \pi_s k_{1s}^* &=& \ 0 \text{ then } b_{2s}^* \in \left[0, \theta A\right], & \text{(40b)}\\
&\text{if } \pi_s\mu_s k_{1s}^* - \pi_s k_{1s}^* &>& \ 0 \text{ then } b_{2s}^* = \theta A. & \text{(40c)}
\end{aligned}$$

Step 2. Using the conditions above I show that, at an optimum, $\mu_s > 1$ and $b_{2s}^* = \theta A$ for $s = l, h$, $q_l^* < q_h^* = 1$, and $k_0^* > 0$ and $k_{1s}^* > 0$ for $s = l, h$. First, notice that conditions (39a)-(39c) imply that

$$\nu_s = \left(\lambda - \mu_s\right)_+ k_0^*. \tag{41}$$

Lemma 3 and no-default in period 2 imply that $G_s'^* - b_{2s}^* > 0$. Then condition (38) can be rewritten as

$$\mu_s \ge \frac{A - b_{2s}^*}{G_s'^* - b_{2s}^*} + \frac{\left(\lambda - \mu_s\right)G_s''^*\left(k_0^* - k_{1s}^*\right) - \left(\lambda - \mu_s\right)_+ G_s''^* k_0^*}{G_s'^* - b_{2s}^*}. \tag{42}$$

Let me show that the expression $\left(\lambda - \mu_s\right)G_s''^*\left(k_0^* - k_{1s}^*\right) - \left(\lambda - \mu_s\right)_+ G_s''^* k_0^*$ is always non-negative. If $\lambda \le \mu_s$ this expression is equal to $\left(\lambda - \mu_s\right)G_s''^*\left(k_0^* - k_{1s}^*\right) \ge 0$, where the inequality follows because either $k_0^* - k_{1s}^* > 0$ and $G_s''^* < 0$ or $k_0^* - k_{1s}^* \le 0$ and $G_s''^* = 0$. If $\lambda > \mu_s$ this expression is equal to $-\left(\lambda - \mu_s\right)G_s''^* k_{1s}^* \ge 0$. Therefore (42) implies that

$$\mu_s \ge \frac{A - b_{2s}^*}{G_s'^* - b_{2s}^*} > 1,$$

where the second inequality follows since $A > 1 \ge G_s'^*$. Condition (40c) then implies that $b_{2s}^* = \theta A$. Since $\mu_s > 0$, the budget constraint is binding in period 1 and

$$k_{1s}^* = \frac{G_s'^* + x_s - b_{1s}}{G_s'^* - \theta A}k_0^*. \tag{43}$$

Proceeding as in the proof of Lemma 4, I can show that $k_{1h}^* - k_0^* > 0$ and $k_{1l}^* - k_0^* < 0$, implying that $q_h^* = G_h'^* = 1$, $q_l^* = G_l'^* < 1$, and $G_h''^* = 0$. Moreover, I can show that k_0^* and k_{1s}^* are positive. First, notice that if $k_0^* = 0$ then (43) implies that the entrepreneur's utility is zero, contradicting the assumption that it is positive. Second, $k_0^* > 0$ and (43) imply that $k_{1s}^* > 0$.

Step 3. Next, I show that $\mu_s > \lambda$ iff $z_{1s}^* > \lambda$. This, together with conditions (39a)-(39c) gives the characterization of $\{b_{1s}^*\}$ in the proposition. Since $k_{1s}^* > 0$ (from step 2), (38) and (42) hold as equalities. Suppose that $\lambda \le \mu_s$. Then, given the definition of z_{1s}^*, subtracting λ on both sides of (42) and rearranging gives

$$\left(\mu_s - \lambda\right)\frac{G_s'^* - \theta A + G_s''^*\left(k_0^* - k_{1s}^*\right)}{G_s'^* - \theta A} = z_{1s}^* - \lambda.$$

Assumption C implies that $G_s'^* - \theta A + G_s''^* (k_0^* - k_{1s}^*) > 0$. Therefore, $\mu_s - \lambda$ has the same sign as $z_{1s}^* - \lambda$. Suppose, instead, that $\lambda > \mu_s$. Then, I obtain

$$(\mu_s - \lambda) \frac{G_s'^* - \theta A - G_s''^* k_{1s}^*}{G_s'^* - \theta A} = z_{1s}^* - \lambda,$$

and, given that $G_s''^* \leq 0$, $\mu_s - \lambda$ inherits the sign of $z_{1s}^* - \lambda$ also in this case.

Step 4. Finally, I prove inequality (22). Since $k_0^* > 0$ (from step 2) (37) must hold as an equality. Using (41) to substitute for ν_s, gives

$$\lambda \left(1 - \sum_s \pi_s b_{1s}^*\right) = \sum_s \pi_s \mu_s (x_s + G_s'^* - b_{1s}^*) - \sum_s \pi_s \left[(\lambda - \mu_s) G_s''^* (k_0^* - k_{1s}^*) - (\lambda - \mu_s)_+ G_s''^* k_0^*\right].$$

Substituting (42) (as an equality), using $q_s^* = G_s'^*$, $G_h''^* = 0$ (from step 3), and the definitions of z_{1s}^* and z_0^* gives

$$(\lambda - z_0^*) \left(1 - \sum_s \pi_s b_{1s}^*\right) = \pi_l \left[(\lambda - \mu_l) G_l''^* (k_0^* - k_{1l}^*) - (\lambda - \mu_l)_+ G_l'' k_0^*\right] \frac{x_l + \theta A - b_{1l}^*}{G_l^* - \theta A}. \tag{44}$$

In step 3 I have shown that $(\lambda - \mu_l) G_l''^* (k_0^* - k_{1l}^*) - (\lambda - \mu_l)_+ G_l'' k_0^* \geq 0$, and it is easy to show that the inequality is strict whenever $\lambda \neq \mu_l$ (notice that $k_0^* - k_{1l}^* < 0$). Since $x_l + \theta A - b_{1l}^* < 0$ and $1 - \sum_s \pi_s b_{1s}^* > 0$, it follows that $\lambda < z_0^*$ except if $\lambda = \mu_l$, in which case $\lambda = z_0^*$. This completes the proof. ■

6.6 Proof of Proposition 3

Before proving the proposition, it is useful to introduce the following lemma.

Lemma 7 *Suppose the planner chooses $\rho^* > \rho^{CE}$ and the associated price is q_l^*, then $q_l^* \leq Q(\rho^*)$ (where the function Q is defined in Lemma 5).*

Proof. Proceeding as in the proof of Lemma 5 define the following function

$$J\left(k_{1l}^T; b_{1l}, b_{1h}\right) \equiv \left(F'\left(k_{1l}^T\right) - \theta A\right) k_{1l}^T + \left(x_l + \theta A - \tilde{b}_{1l}\right) k_0,$$

where

$$\begin{aligned}
\tilde{b}_{1l} &= \min\left\{b_{1l}, \theta a_l + F'\left(k_{1l}^T\right) - \gamma\right\} \\
k_0 &= \frac{1}{1 - \sum_s \pi_s b_{1s}} \left[n + 3e + \pi_l \left(F\left(k_{1l}^T\right) - F'\left(k_{1l}^T\right) k_{1l}^T\right) - U^{CE}\right].
\end{aligned} \tag{45}$$

Suppose the planner chooses the total borrowing ratio ρ^*, and the associated borrowing ratios $\{b_{1l}^*, b_{1h}^*\}$, and let q_l^* and k_{1l}^{T*} be the associated equilibrium price and quantity on the used capital market. By definition, $J\left(k_{1l}^{T*}; b_{1l}^*, b_{1h}^*\right) = 0$. Moreover, the definition of U^{CE} implies that

$$3e + \pi_l \left(F\left(k_{1l}^{T,CE}\right) - F'\left(k_{1l}^{T,CE}\right) k_{1l}^{T,CE}\right) = U^{CE},$$

and, by construction, it follows that $J\left(k_{1l}^{T,CE}; b_{1l}^{CE}, b_{1h}^{CE}\right) = H\left(k_{1l}^{T,CE}; b_{1l}^{CE}, b_{1h}^{CE}\right) = 0$. Differentiating the function J and noticing that $\partial \tilde{b}_{1l} / \partial k_{1l}^T \leq 0$, gives

$$\frac{\partial J}{\partial k_{1l}^T} \geq F'\left(k_{1l}^T\right) - \theta A + \left[1 - \pi_l \frac{x_l + \theta A - \tilde{b}_{1l}}{1 - \sum_s \pi_s b_{1s}}\right] F''\left(k_{1l}^T\right) k_{1l}^T.$$

Some algebra shows that η (as defined in Assumption C') is an upper bound for the expression in square brackets on the right-hand side. Therefore, Assumption C' is sufficient to ensure that J is monotone increasing in k_{1l}^T. Moreover, it is possible to show that the function J is monotone decreasing in b_{1l} and b_{1h}. If $\rho^* > \rho^{CE}$, it follows that $b_{1l}^* \geq b_{1l}^{CE}$ and $b_{1h}^* \geq b_{1h}^{CE}$. Therefore, $J\left(k_{1l}^{T*}; b_{1l}^{CE}, b_{1h}^{CE}\right) \geq 0$. Given that $J\left(k_{1l}^{T,CE}; b_{1l}^{CE}, b_{1h}^{CE}\right) = 0$, the last inequality implies that $k_{1l}^{T*} \geq k_{1l}^{T,CE}$. Since the profits of the traditional sector, $F\left(k_{1l}^T\right) - F'\left(k_{1l}^T\right) k_{1l}^T$, are increasing in k_{1l}^T, it follows that

$$3e + \pi_l \left(F\left(k_{1l}^{T,*}\right) - F'\left(k_{1l}^{T,*}\right) k_{1l}^{T,*}\right) - U^{CE} \geq 0. \tag{46}$$

Comparing (36) and (45), using inequality (46), shows that $H\left(k_{1l}^{T,*}; b_{1l}^*, b_{1h}^*\right) \geq 0$. Let $\hat{k}_{1l}^T$ denote the value of k_{1l}^T that solves $H\left(k_{1l}^T; b_{1l}^*, b_{1h}^*\right) = 0$, where H is the function defined in Lemma 5. Since H is monotone in k_{1l}^T it follows that $\hat{k}_{1l}^T \leq k_{1l}^{T*}$, which implies that $q_l^* = F'\left(k_{1l}^{T*}\right) \leq F'(\hat{k}_{1l}^T) = Q\left(\rho^*\right)$. ■

Now, I can turn to Proposition 3. I will proceed by contradiction, assume that $\rho^* > \rho^{CE}$, and show that this leads to a violation of the optimality conditions of the planner's problem. Notice that when $\bar{U} = U^{CE}$, the entrepreneurs must achieve positive utility at the social optimum (since they do so at the competitive equilibrium, and the competitive allocation is feasible). So Proposition 2 applies.

Step 1. I first define the values $\hat{z}_0$ and $\hat{z}_{1s}$, which will be used below. Let $\hat{q}_h$ and $\hat{q}_l$ denote the prices which would arise in competitive equilibrium if the entrepreneurs were to choose $\{b_{1s}^*\}$ instead of $\left\{b_{1s}^{CE}\right\}$. That is, $\hat{q}_h = 1$ and $\hat{q}_l = Q\left(\rho^*\right)$, where $Q\left(.\right)$ is the function defined in Lemma 5. Let $\hat{z}_0$ and $\hat{z}_{1s}$ denote the values of z_0 and z_{1s} obtained substituting $\{b_{1s}^*\}$ and $\{\hat{q}_{1s}\}$ in (14) and (15). Notice that substituting $\{b_{1s}^*\}$ and $\{q_{1s}^*\}$ in (14) and (15) gives z_0^* and z_{1s}^*.

Step 2. I now derive some inequalities regarding $\hat{z}_0$ and $\hat{z}_{1s}$. It is possible to show that $\rho^* > \rho^{CE}$ implies $q_l^* \leq \hat{q}_l$. This follows from Lemma 7, and here is where Assumption C' is used. This result immediately implies that $\hat{z}_{1l} \leq z_{1l}^*$. Moreover, $q_h^* = \hat{q}_h = 1$ implies that $\hat{z}_{1h} = z_{1h}^*$. Finally, I want to show that $\hat{z}_0 \geq z_0^*$. To prove this inequality, it is sufficient to show that (15) defines an increasing function in q_l, which follows from differentiating (15) with respect to q_l and using the fact that $x_l + \theta A - b_{1l} < 0$.

Step 3. Define the state s' as follows: $s' = h$ if $0 < \rho^* \leq \hat{\rho}$ and $s' = l$ if $\rho^* > \hat{\rho}$ ($\rho^* = 0$ is not possible since $\rho^* > \rho^{CE} \geq 0$ by hypothesis). The construction in step 1 implies that

$$\hat{z}_{1s'} - \hat{z}_0 = \zeta\left(\rho^*\right),$$

where $\zeta\left(.\right)$ is the function defined in Lemma 6. Since $\rho^* > \rho^{CE}$, Corollary 1 implies that $\zeta\left(\rho^*\right) > 0$. Finally, Proposition 2 shows that $\lambda^* \leq z_0^*$. Putting together these two inequalities and those derived in step 2, gives

$$z_{1s'}^* - \lambda^* \geq z_{1s'}^* - z_0^* \geq \hat{z}_{1s'} - \hat{z}_0 > 0.$$

Notice that the definition of the state s' implies that $b_{1s'}^* > 0$. The inequalities $b_{1s'}^* > 0$ and $z_{1s'}^* - \lambda^* > 0$ violate the planner's optimality conditions derived in 2. This completes the argument.

Finally, I turn to the last statement of the proposition. Suppose that the equilibrium is of type 1, and suppose, by contradiction, that $\rho^* \geq \rho^{CE}$. Then, steps analogous to the ones above lead to the chain of inequalities

$$z_{1s'}^* - \lambda^* > z_{1s'}^* - z_0^* \geq \hat{z}_{1s'} - \hat{z}_0 \geq 0,$$

where the first inequality is now strict, because of Proposition 2. Again, I obtain a contradiction. ■

Lemma 8 *Consider the efficient allocation in Proposition 3. The corresponding financial contract satisfies*

$$d_0^* \leq \sum \pi_s b_{1s}^* k_0^*.$$

Proof. Constraint (19) can be rewritten as

$$-d_0^* + \sum_s \pi_s b_{1s}^* k_0^* \geq U^{CE} - 3e - \sum_s \pi_s \left(F\left(k_{1s}^{T*}\right) - q_s k_{1s}^{T*}\right). \tag{47}$$

Moreover, Proposition 3 shows that $\rho^* \leq \rho^{CE}$. An argument symmetric to the one used in the previous proof shows that $\rho^* \leq \rho^{CE}$ implies that $k_{1l}^{T*} \leq k_{1l}^{T,CE}$, and thus

$$3e + \pi_l \left(F\left(k_{1l}^{T,*}\right) - F'\left(k_{1l}^{T,*}\right) k_{1l}^{T,*}\right) - U^{CE} \leq 0. \tag{48}$$

Putting together (47) and (48) gives the desired result. ■

6.7 Full commitment on the entrepreneurs' side

Here, I discuss the equilibrium in the case where entrepreneurs have unlimited ability to commit future repayments, i.e., when $\theta = 1$. Let me derive first the first best allocation, next I will show that this allocation can be achieved in equilibrium without violating the consumers' participation constraints. Consider the planner's problem

$$\max \sum_s \pi_s c_{2s}^e,$$

subject to

$$\begin{aligned} & k_0 \leq n + d_0, \\ & q_s k_{1s} \leq (q_s + x_s) k_0 - d_{1s} \quad \text{for } s = l, h, \\ & c_{2s}^e \leq A k_{1s} - d_{2s} \quad \text{for } s = l, h, \end{aligned}$$

and

$$\begin{aligned} & e - d_0 + \sum_s \pi_s \left(e + d_{1s} - k_{1s}^T + e + d_{2s} + F\left(k_{1s}^T\right)\right) \geq \bar{U}, \\ & e - d_0 \geq 0, \quad e + d_{1s} - k_{1s}^T \geq 0, \quad d_{2s} + F\left(k_{1s}^T\right) \geq 0. \end{aligned}$$

with $\bar{U} = 3e$. Note that in this case it is necessary to take into account the non-negativity constraints for the consumers' consumption levels. It is possible to show that the optimum is achieved when

$$\begin{aligned} d_0 &= e, \quad d_{1s} = -e, \quad d_{2s} = 2e, \\ c_{2s}^e &= A(1 + x_s)(n + e) + Ae - 2e, \\ k_0 &= n + e, \quad k_{1s} = (1 + x_s)(n + e) + e, \quad k_{1s}^T = 0. \end{aligned}$$

The same allocation is achieved in a competitive equilibrium with prices $q_h = q_l = 1$. Given that these prices are constant at 1 in a neighborhood of the planner's optimum, it is easy to show that the first-order conditions of the individual problem are satisfied at the planner's optimum. Moreover, absent the no-default conditions of the entrepreneur, the individual problem is concave so the planner's optimum is also an individual optimum.

6.8 Proof of Proposition 4

The entrepreneur's problem is now to maximize

$$\sum_s \pi_s (A - b_{2s}) k_{1s},$$

subject to

$$
\begin{aligned}
k_0 &\leq \nu n && (49)\\
k_0 &\leq n + (\sum_s \pi_s b_{1s}) k_0 - \tau_0,\\
q_s k_{1s} &\leq (q_s + x_s - b_{1s})\, k_0 + b_{2s} k_{1s} \quad \text{for } s = l, h,
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq b_{1s} \leq \theta a_s + q_s - \gamma \quad \text{for } s = l, h,\\
0 &\leq b_{2s} \leq \theta A \quad \text{for } s = l, h.
\end{aligned}
$$

The first order conditions are the same as in those derived in Lemma 2, with the exception of that for k_0 which gives

$$z_0 = \frac{\sum_s \pi_s z_{1s} (x_s + q_s - b_{1s}) - \xi}{1 - \sum_s \pi_s b_{1s}},$$

where ξ is the Lagrange multiplier on (49). Take a constrained efficient allocation, characterized in 2, set $\nu = n/k_0^*$ and

$$\tau_0 = 3e - \sum_s \pi_s \left(G\left(k_0^* - k_{1s}^*\right) - G'\left(k_0^* - k_{1s}^*\right)\left(k_0^* - k_{1s}^*\right)\right) - \bar{U}.$$

To show that $\{b_{1s}^*\}$ solves the entrepreneur's optimization substitute in the entrepreneur's first order conditions, setting $z_0 = \lambda^*$ and ξ equal to the right-hand side of (44). ■

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